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Kalman Filter Predictor and Initialization Algorithm for PRI Tracking

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A pulse repetition interval (PRI) tracking algorithm has been developed for staggered PRI sequences. The algorithm is intended for use in electronic countermeasures applications. It processes time of arrival (TOA) measurements from an electronic support receiver and outputs TOA predictions with associated variances to an electronic attack system. The algorithm uses a Kalman filter for prediction combined with a preprocessing routine to determine the period of the stagger sequence and to construct an uncorrupted data set for Kalman filter initialization. It is robust to missing and/or spurious pulses, intentional or unintentional jitter, and measurement noise.

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KALMAN FILTER PREDICTOR AND INITIALIZATION
ALGORITHM FOR PRI TRACKING

INTRODUCTION

The problem considered in this report is pulse repetition interval (PRI) tracking. PRI is the time interval between two pulses emitted by a radar. A radar may have a constant PRI, or it may have some form of PRI agility, in which case the time interval between pulses varies on a pulse-to-pulse basis. Currently, a large number of anti-ship missiles use radar as the homing device. Some currently have PRI agility. However, future threats in this category are expected to be both PRI and frequency agile.

A PRI tracking algorithm is needed as the basis for a deceptive countermeasures technique. When an emitter has been selected for deceptive countermeasures, the PRI tracker should predict the PRI or, equivalently, the pulse time of arrival (TOA) of the next pulse so that a jamming pulse can be gated on at that time. In addition to predicting the next pulse TOA, the algorithm should also generate a measure of the variance of the prediction. The variance of the prediction, along with the measured pulse width, is used to control the width of the jamming pulse. It is important to keep the jamming pulse length as short as possible. When the variance of the prediction is small, the jamming pulse length approaches the received pulse width. This allows two significant operational advantages. First, the ECM transmitter can be time multiplexed to handle multiple incoming threats simultaneously. Second, ownship RFI problems are minimized.

The three most common types of PRI agility are staggered PRIs, sinusoidally modulated PRIs, and PRIs with random jitter [1]. This report considers the staggered PRI case. Additional work is currently underway to develop algorithms to handle the other cases, and this will be the topic of a future report. A staggered PRI is a sequence of several different pulse intervals in a repeating pattern. For example, the sequence {211,400,400,400,315,211,400,400,400,315,...} has three distinct pulse intervals and a period of five. It is referred to as a three-element, five-position stagger with stagger elements of 211, 400, and 315. Figure 1 illustrates this pulse train.

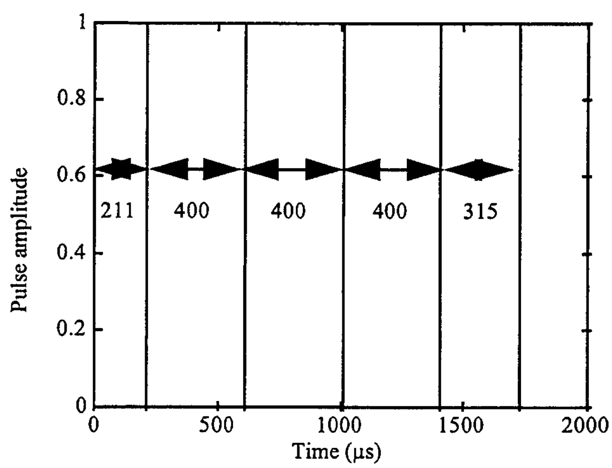

Fig. 1 — Three-element, five-position staggered pulse train

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ASSUMPTIONS

In designing the algorithm, the following assumptions were made:

1. Emitters of concern are specified by the electronic attack (EA) system for tracking.

2. For each specified emitter, a group of pulses is accumulated in a buffer for initialization.

3. An electronic support (ES) receiver has sorted PRI measurements so that all measurements in a given buffer will, in theory, come from the same emitter. However, since all ES systems make mistakes, the sorting process is not perfect. As a result, missing and/or spurious pulses as well as jitter and measurement noise may corrupt the PRI sequence in the buffer.

4. After initialization, pulses are received at the per-pulse data rate of 100 to 10,000 pulses per second.

5. The per-pulse data stream contains pulses from a single emitter, but missing and/or spurious pulses, jitter, and measurement noise may corrupt the data stream.

DATA SIMULATION

Because no measured per-pulse data were available, all of the data used for testing was simulated. Simulated data consist of TOAs and PRIs written to a file. Two PRI sequences were used for testing: a 32-position, 16-level stagger sequence, and a 14-position, 5-level stagger sequence.

Because measurements received from the ES system will be corrupted by jitter and missing pulses, it was necessary to model these effects in the simulated data. Jitter was accounted for using the following cumulative model [2]:

$$
\begin{aligned}
TOA_j &= TOA_{j-1} + PRI_j + w_j && w_j \sim N(0,\sigma_w^2) \\
\tau_j &= TOA_j + v_j && v_j \sim N(0,\sigma_v^2),
\end{aligned}
$$

where TOA_j is the time of arrival of the jth pulse and τ_j is the measured TOA. The white noise term w_j includes the effects of oscillator instability and deceptive jitter, which are added to the data at the emitter. The white noise term v_j simulates the effect of measurement noise, which is added to the data by the receiver. To account for missing pulses that occur when the probability of detection is less than one or when the ES deinterleaver makes a mistake, different data sets were generated with between zero and 25% missing pulses. A single missing pulse is represented in the data by a missing PRI value followed by an unusually large PRI value. The large value is the sum of the missing PRI and the PRI that follows it.

FOURIER REPRESENTATION OF STAGGERED PRI SEQUENCE

A staggered PRI sequence can be viewed as a discrete time series in which PRIs are plotted vs equally spaced pulse indices. A staggered PRI sequence is periodic, and each period contains an integer number of pulses. The sequence repeats in a deterministic manner.

The Fourier representation theorem states that any periodic function can be expressed as a linear combination of sine and cosine terms plus a constant. The frequencies of the sine and cosine terms correspond to the different harmonics present in the function.

A discrete time series can be represented by a finite number of harmonics. Specifically, if a discrete time series has period $= p$, the first harmonic has frequency $1/p$ and completes its cycle in p time periods. The second harmonic has frequency $2/p$ and completes its cycle in $p/2$ time periods. If p is even, at most $p/2$ harmonics are required to represent the time series because the period corresponding to the $(p/2)$th harmonic is 2, which is the shortest possible cycle length. If p is odd, at most $(p\text{-}1)/2$ harmonics are needed [3].

Therefore, a staggered PRI sequence can be represented exactly as follows:

$$\begin{aligned} PRI(k) = \overline{PRI} &+ a_1 \cos\left(2\pi \frac{1}{p} k\right) + b_1 \sin\left(2\pi \frac{1}{p} k\right) \\ &+ a_2 \cos\left(2\pi \frac{2}{p} k\right) + b_2 \sin\left(2\pi \frac{2}{p} k\right) \\ &+ \\ &\vdots \\ &+ a_m \cos\left(2\pi \frac{m}{p} k\right) + b_m \sin\left(2\pi \frac{m}{p} k\right), \end{aligned}$$

where

$\overline{PRI}$ is the mean PRI

k is the index of the kth pulse

p is the period of the sequence

a_i, s, and b_i s are the Fourier coefficients, and

$$m = \begin{cases} p/2 & \text{for } p \text{ even} \\ (p-1)/2 & \text{for } p \text{ odd}. \end{cases}$$

The last sine term evaluates to zero when p is even.

KALMAN FILTER MODEL FOR PRI SEQUENCE WITH KNOWN PERIOD

If the period of the staggered PRI sequence can be determined with preprocessing, then the following Kalman filter model can be used to predict PRIs:

$$\text{System equation:} \qquad \theta_t = \theta_{t-1} + w_t \quad w_t \sim N(0, \sigma_w^2),$$

$$\text{Observation equation:} \quad Y_t = F_t'\theta_t + v_t \quad v_t \sim N(0, \sigma_v^2),$$

where

θ_t is the state vector,

F_t is the measurement matrix,

Y_t is the measured PRI,

w_t is the system noise, and

v_t is the measurement noise.

The length of the state vector is equal to the period of the PRI sequence. Thus a 64-position stagger sequence requires a Kalman filter of order 64. The state vector contains the mean PRI followed by the Fourier coefficients for the PRI sequence. The measurement matrix is time-varying. It contains a 1 followed by cosine and sine terms for all of the harmonics present in the PRI sequence. As an example, the state vector and measurement matrix for a PRI sequence with period = 5 is

$$\theta_t' = \left[\overline{PRI} \ \ a_1 \ \ b_1 \ \ a_2 \ \ b_2 \ \right],$$
$$F_t' = \left[1 \ \cos\!\left(2\pi\tfrac{1}{p}t\right) \ \sin\!\left(2\pi\tfrac{1}{p}t\right) \ \cos\!\left(2\pi\tfrac{2}{p}t\right) \ \sin\!\left(2\pi\tfrac{2}{p}t\right)\right].$$

The filter converges after processing one complete period of the PRI sequence if the first period of data does not have any missing or spurious pulses in it.

Figure 2 illustrates the convergence of the Kalman filter on the 14-position, 5-level stagger sequence. Circles represent actual PRIs, and the solid line represents predicted PRIs. Since the period of the sequence is 14, the filter converges after processing the first 14 PRIs.

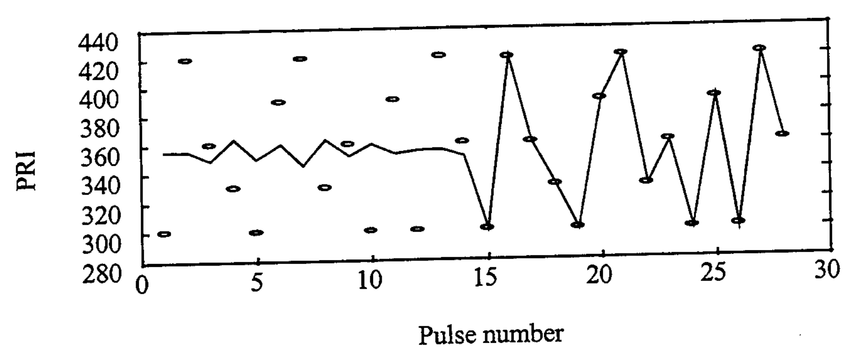

Fig. 2 — Kalman filter convergence for 14-position, 5-level stagger (system noise = 1.0, measurement noise = 0.5, no missing pulses)

Figure 3 illustrates the convergence of the Kalman filter on the 32-position, 16-level stagger sequence. Since the period of the sequence is 32, the filter converges after processing the first 32 PRIs.

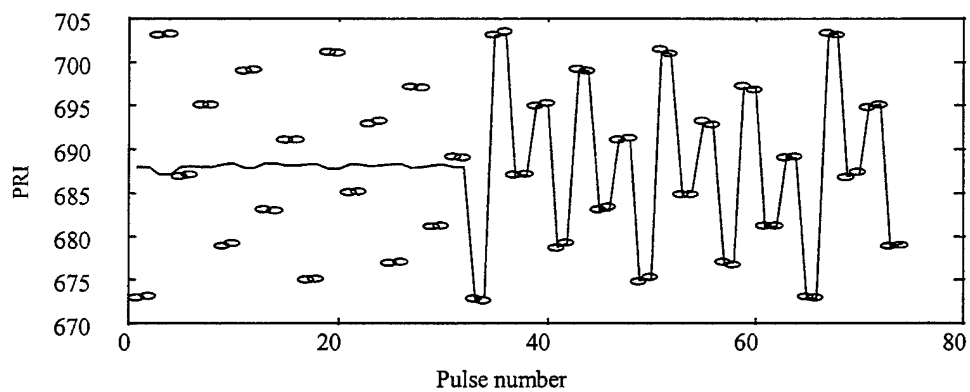

Fig. 3 — Kalman filter convergence for 32-position, 16-level stagger (system noise = 0.1, measurement noise = 0.05, no missing pulses)

Specification of System Noise and Measurement Noise

By definition, system noise is noise that is added to the data at the emitter. This type of noise includes deceptive jitter and oscillator instability. Measurement noise is added to the data at the receiver. However, in this application, all noise on the data can be treated as measurement noise, and the system noise variance in the Kalman filter can be set to a very small value. The following paragraphs provide an explanation of why this can be done.

In a Kalman filter model, the observation at time t contains system noise that is integrated from time zero to time t. The following simple example illustrates this:

$$\text{System equation:} \qquad x_t = x_{t-1} + w_t \quad w_t \sim N(0, \sigma_w^2)$$

$$\text{Observation equation:} \quad y_t = x_t + v_t \qquad v_t \sim N(0, \sigma_v^2)\,.$$

Therefore, the system equation propagates as

$$\begin{aligned} x_1 &= x_0 + w_1 \\ x_2 &= x_0 + w_1 + w_2 \\ &\vdots \\ x_t &= x_0 + \sum_{i=1}^{t} w_i \ . \end{aligned}$$

By using this expansion at each time t, the observation equation is given by

$$y_t = x_0 + \sum_{i=1}^{t} w_i + v_t \ .$$

Since the system noise is integrated and the measurement noise is not integrated, it is necessary to distinguish system noise from measurement noise and specify them correctly.

Similarly, in this application, the TOA measurements contain integrated system noise. The following cumulative jitter model is assumed for the TOAs when they are generated:

$$
\begin{aligned}
TOA_j &= TOA_{j-1} + PRI_j + w_j \\
TOA_1 &= TOA_0 + PRI_1 + w_1 \\
TOA_2 &= TOA_0 + PRI_1 + PRI_2 + w_1 + w_2 \\
&\vdots \\
TOA_n &= TOA_0 + \sum_{i=1}^{n} PRI_i + \sum_{i=1}^{n} w_i .
\end{aligned}
$$

The TOAs measured at the receiver have the form

$$measured\_TOA_n = TOA_0 + \sum_{i=1}^{n} PRI_i + \sum_{i=1}^{n} w_i + v_n .$$

In this application, the Kalman filter processes PRIs rather than TOAs. PRIs are computed as the difference between successive TOAs:

$$
\begin{aligned}
measured\_PRI_n &= measured\_TOA_n - measured\_TOA_{n-1} \\
&= PRI_n + w_n + v_n - v_{n-1} \\
Var(measured\_PRI_n) &= \sigma_w^2 + 2 * \sigma_v^2 .
\end{aligned}
$$

Since the measured PRIs do not contain integrated system noise, it is not necessary to distinguish between system noise and measurement noise in the Kalman filter model. The system noise variance in the Kalman filter is set to a small value to prevent the filter gain from going to zero. The remaining noise on the data can be treated like measurement noise. The measurement noise variance is estimated on line using Bayesian variance learning, as discussed below.

Variance Learning

The Kalman filter can be modified to incorporate variance learning [4,5]. Variance learning is a Bayesian technique that is used to determine an unknown measurement noise variance from the data. The derivation relies on normal-gamma conjugate theory from which it is known that if a prior distribution is gamma and data are obtained from a normal distribution, then the posterior distribution will be gamma.

A gamma distribution is assumed for the inverse of the unknown measurement noise variance. Data are obtained in the form of prediction errors that have a normal distribution. The posterior distribution for the inverse of the measurement noise variance is therefore gamma. As each pulse is received, the parameters of this gamma distribution are updated. The inverse of the mean of this distribution is used as an estimate of the measurement noise variance.

In this application, variance learning is especially useful since almost all of the noise on the data can be treated like measurement noise. The variance of the prediction depends on both system noise and measurement noise. Since system noise is very small in this case, the variance of the prediction depends primarily on measurement noise that is determined from the data. The Kalman filter therefore provides an accurate value for the variance of the prediction, even when the amount of jitter on the data is not known ahead of time. This is important for sizing the jam gate correctly.

Figure 4 illustrates variance learning on the 14-position, 5-level stagger sequence. Jitter with a variance of 1.0 and measurement noise with a variance of 0.5 was added to the TOA values when the data were generated. The total variance on the PRI values is 1.0 + 2*0.5 = 2. The variance of the prediction approaches the correct value of 2.0.

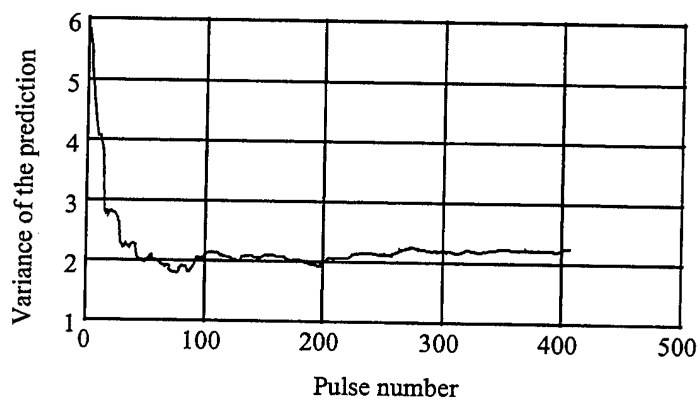

Fig. 4 — Measurement noise variance learning for sequence with total noise variance = 2

Compensation for Missing or Spurious Pulses

It is a requirement that the Kalman filter make accurate predictions of the true PRI sequence in an environment with up to 25% missing pulses and possible spurious pulses. Missing pulses result in PRIs that are much larger than actual PRIs in the sequence, while spurious pulses result in PRIs that are much smaller than actual PRIs. These false PRIs produce very large errors if they are fed directly into the Kalman filter. The problem is illustrated in Fig. 5 for the 14-position, 5-level stagger sequence. In this case, a single missing pulse was inserted into the data stream at about pulse number 25. It can be seen that this missing pulse is still causing significant prediction errors 100 pulses later.

To compensate for missing or spurious pulses, the following procedure is used. At each iteration, the Kalman filter makes a prediction with an associated variance. If the next received PRI does not fall within four standard deviations of the prediction, it is assumed to be a false PRI. It is not used to update the filter, and the next prediction is adjusted to compensate for the false PRI. For missing pulses (received PRI greater than the current prediction plus four standard deviations), the next prediction is the sum of two PRIs, and this cumulative prediction has an increased variance associated with it [3,4]. For spurious pulses (received PRI less than the current prediction minus four standard deviations), the next prediction is the difference between the current prediction and the false PRI. The variance of this prediction is the same as the current prediction. Figure 6 shows the Kalman filter performance for the

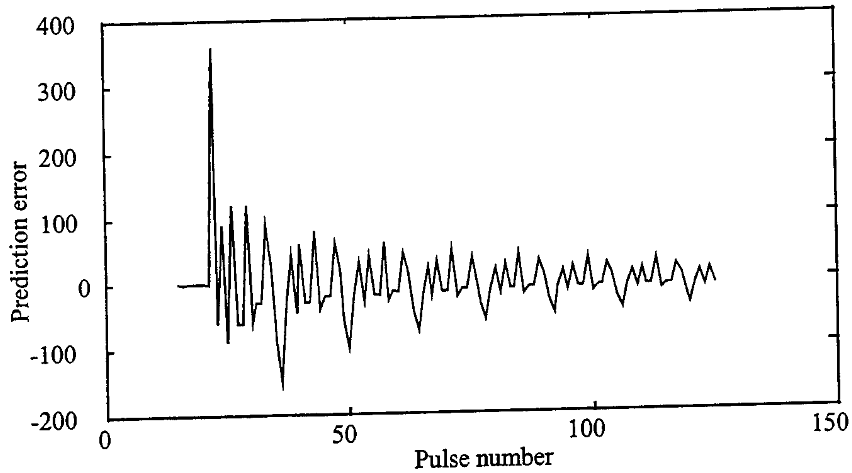

Fig. 5 — Effect of a single missing pulse on prediction error (system noise = 1.0, measurement noise = 0.5)

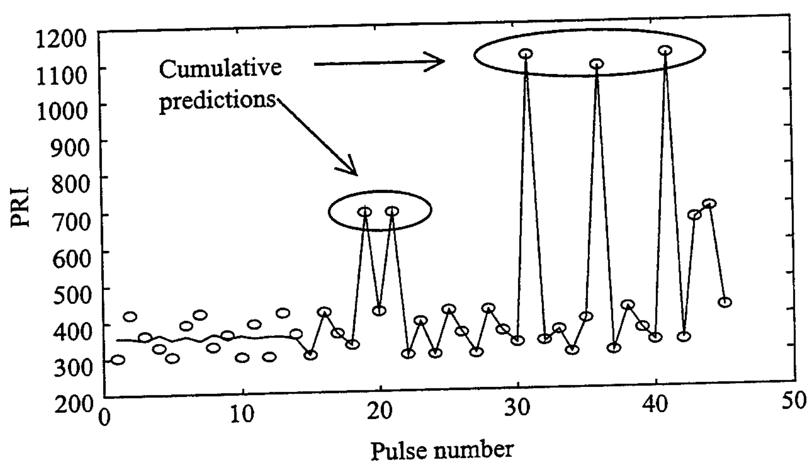

Fig. 6 — Performance of Kalman filter with compensation for missing pulses (25% missing pulses, system noise = 1.0, measurement noise = 0.5)

14-position, 5-level stagger sequence with 25% of the pulses missing. Measured PRIs of about 700 occur when a single pulse is missing. Measured PRIs of about 1100 occur when two successive pulses are missing. The initialization algorithm described in the following section fills in pulses that are missing from the first period.

INITIALIZATION ALGORITHM

A preprocessing routine is needed to determine the period of the PRI sequence and to construct one complete period of uncorrupted data for filter initialization. It is assumed that a buffer of PRI measurements is available for preprocessing.

Although the goal is to determine the number of pulses per period, it is easier to first find the period of the sequence in milliseconds. This is because the number of pulses per period will vary when there are missing or spurious pulses, but missing or spurious pulses do not affect the length of the period in milliseconds.

Once the period in milliseconds has been determined, the entire length of data in the buffer can be divided into periods and the periods overlayed. A pulse that is missing in one period will not be missing in all periods if enough data are available for preprocessing. From the overlayed data, the number of pulses per period as well as the sequence of PRI values can be identified.

Searching for the Period

If the period of a PRI sequence is *T* ms, then every PRI in a data set will repeat in approximately *T* ms with some variability due to jitter. Therefore, when there are no missing pulses, every PRI in a data set can provide an estimate of the period *T*. Starting successively at each PRI in the data set, the estimates are obtained by searching forward for similar PRI values in the sequence. When a similar PRI is found, the intervening PRIs are added to obtain an estimate of *T*. Figure 7 illustrates this search procedure.

The forward search procedure will yield some estimates of *T* that are incorrect, as well as the correct estimates of *T* at each PRI. This is because the same PRI can appear more than once within a single period. However, the number of estimates at the correct value will always exceed the number of estimates at any given incorrect value. Similar estimates of *T* are grouped together in bins. The bin with the maximum number of estimates contains the correct value for *T*. Figure 8 shows the results of binning for the 14-position, 5-level stagger sequence with no missing pulses. The sequence has a period of approximately 5 ms. Figure 9 presents the results of binning for the same sequence with 25% missing pulses.

It is necessary to put a check into the algorithm to ensure that a multiple of *T* is not identified as the period of the sequence rather than *T* itself. This can happen because any sequence with period *T* also has period 2*T*, 3*T*, etc. Bins centered on multiples of *T* contain large numbers of estimates as well as the bin centered on *T*.

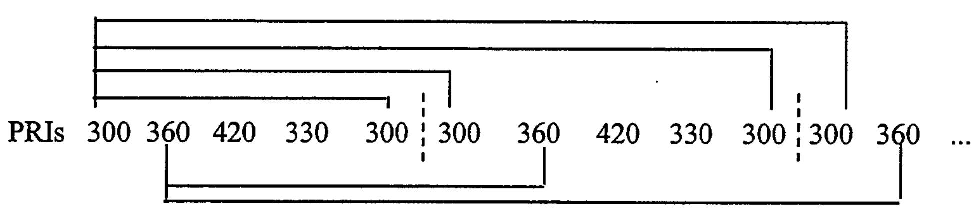

Fig. 7 — Search procedure for possible periods

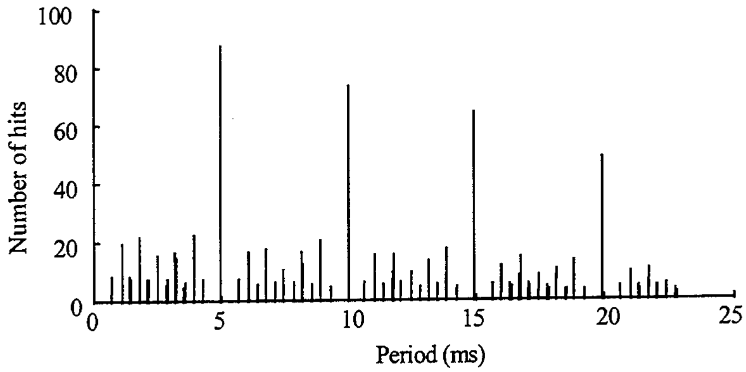

Fig. 8 — Results of binning for a sequence with no missing pulses (eight periods of data, 14-position, 5-level stagger sequence)

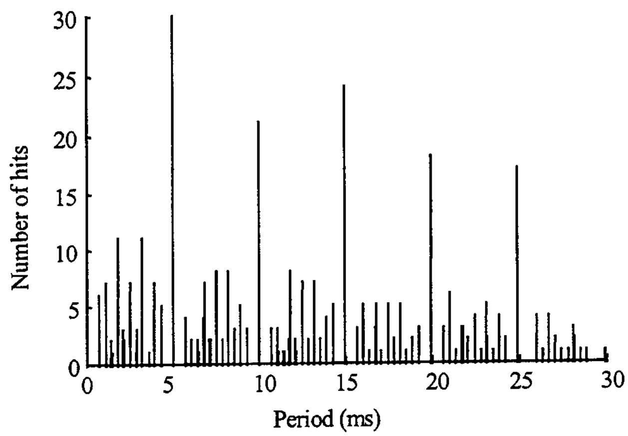

Fig. 9 — Results of binning for a sequence with 25% missing pulses (eight periods of data, 14-position, 5-level stagger sequence)

Determining the Sequence

The PRI sequence is determined by first dividing the TOA data in the buffer into periods, using knowledge of T. A search procedure is used to find the first TOA in each period. This procedure takes into account the increasing variance of the TOAs from the beginning to the end of each period.

Next, the periods are overlayed. TOA values beyond the first period are converted to values relative to the start of the period in which they are contained. Similar TOA values from each period are then grouped together in bins and averaged. Figure 10 illustrates the results of binning TOAs for the 14-position stagger sequence with no missing pulses and eight periods of data. Figure 11 shows the results of binning for the same sequence with 25% of the pulses missing. Finally, the sequence of PRIs is obtained by taking differences between the consecutive, averaged TOAs.

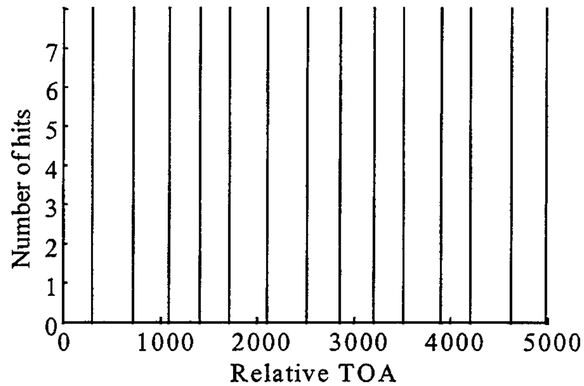

Fig. 10 — Results of binning the relative TOAs with no missing pulses (eight periods of data, 14-position, 5-level stagger sequence)

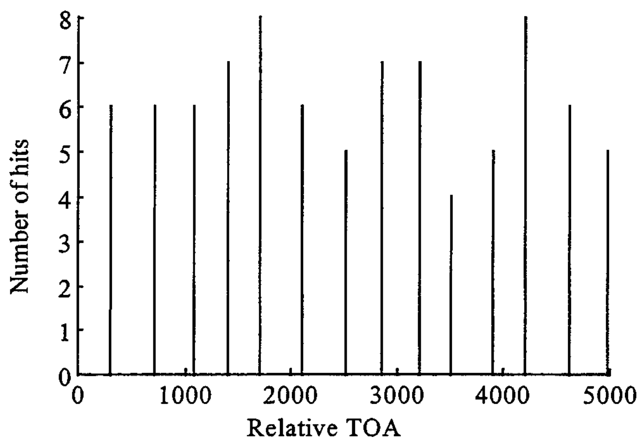

Fig. 11 — Results of binning the relative TOAs with 25% missing pulses (eight periods of data, 14-position, 5-level stagger sequence)

REQUIRED QUANTITY OF DATA

The quantity of data required for the initialization algorithm to be successful depends on the percentage of missing pulses and the period of the stagger sequence. It is desirable to have the algorithm work with up to 25% missing pulses and with periods up to length 64.

A rough estimate of the quantity of data required can be obtained by computing the number of periods of the sequence needed to ensure that at least two PRI values are measured in each stagger position. If the probability of detection equals one, two periods of the sequence are required. If the probability of detection is less than one, the probability of measuring two or more pulses in each stagger position is computed using the binomial distribution. For a 64-position stagger sequence, the probability is given by

$$prob = \left(1 - \binom{n}{0} (p_d)^0 (1-p_d)^n - \binom{n}{1} (p_d)^1 (1-p_d)^{n-1}\right)^{64}$$

where n is the number of periods of the sequence, and
p_d is probability of detection.

This expression is plotted in Fig. 12. From the figure it can be seen that, to ensure a high probability of receiving two or more measurements in every stagger position, nine periods of data are required when the probability of detection is 0.75 and six periods of data are required when the probability of detection is 0.90.

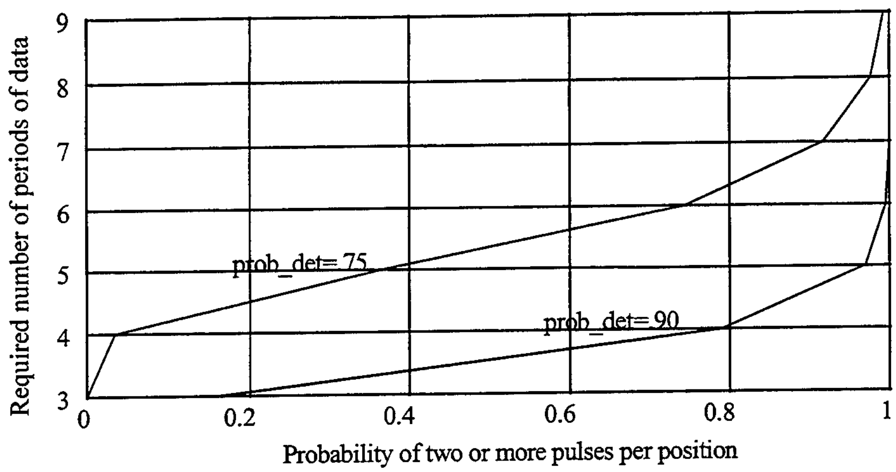

Fig. 12 — Required quantity of data for initialization of a 64-position stagger sequence

SUMMARY

An algorithm for tracking staggered PRI sequences has been developed. The algorithm has two parts: a preprocessing routine, and a Kalman filter model based on the Fourier representation of the staggered PRI sequence. The preprocessing routine determines the period of the stagger sequence and constructs an uncorrupted period of data for Kalman filter initialization. The Kalman filter predicts pulse TOAs and generates the variance of each prediction using a Bayesian variance learning technique.

The algorithm was tested using simulated data. Jitter, measurement noise, and random missing pulses were incorporated into the simulated data sets. Both the preprocessing routine and the Kalman filter performed well with up to 25% missing pulses.

In data sets with up to 25% missing pulses distributed randomly throughout the data set, usually not more than two or three adjacent pulses will be missing. Future work will address longer runs of missing pulses caused by multipath fading. It is important for the algorithm to continue working without reinitialization when fade periods are as long as 0.25 s. Conditions under which a track should be dropped will also be investigated. Finally, this report considered only staggered PRI modulation. Other types of modulation, including sinusoidal and random, will be addressed in a future report.

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